Planning elective surgeries with detailed bed leveling

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Abstract

In the literature on surgical planning, several models focus solely on the operating theatre, minimizing cost functions related to inefficiency (e.g. room under-utilization, surgeon overtime), or to patient-centered quality indices (e.g. time in the waiting list). While the operating theatre is the core of the surgical path, the integration of OR scheduling decisions with those at other stages of such path can significantly affect the impact and the viability of a given surgical plan. This paper presents a surgical planning model in which OR schedule and detailed bed occupancy are coordinated. Given the waiting lists of different surgical disciplines, the model determines a weekly plan for elective surgeries that accounts for different levels of intensity of care required by the patients, skilled surgeon availability, number of beds in the wards, post surgical expected length of stay, bed requirements and patient gender. The model has been tested on real data from a medium-size Italian public hospital. Thorough tests have been performed, for different hospital configurations and management policies, showing the practical viability of our approach.

Keywords: Health service, surgical planning, operating room scheduling, bed levelling, care level

1. Introduction

Operating rooms (ORs) are a critical resource and a main driver of the quality and costs of health services in hospitals [21, 10]. Typically, several ORs, possibly with different characteristics, are organized/managed into a single Operating Theatre (OT), and a single OT is usually shared among different *surgical disciplines*. In the so-called *block scheduling* approach, OR time is divided into *OR sessions*, i.e., time intervals during which an OR is allocated to a single discipline. In this context, the two most studied OT management problems are *(i)* the Master Surgical Schedule Problem (MSSP) and *(ii)* the Surgical Case Assignment Problem (SCAP). The former problem deals with the definition of an assignment of surgical disciplines to OR sessions and it is typically a tactical level problem since it usually has to be solved every few months [9]. The latter is an operational problem that focuses on a short time horizon (usually a week) and deals with the assignment of elective surgeries to OR sessions [22].

Typically, both decision problems focus on maximizing OR occupancy, under constraints related to the quality of service as well as to the schedule of involved personnel [19]. However, operating rooms may not be the only bottleneck in the surgical pathway, since often post surgery beds can cause problems and hinder the patients flow. Poor surgical planning may result in *bed shortage*, i.e., no beds are temporarily available to host a surgical patient. In these cases, either the surgical schedule has to be rearranged (thus causing obvious discomforts), or the managers must face exceptional decisions such as resorting to beds from external units (e.g., a different ward), or even early discharging with increased risk for the patients. In any case, such real-time adjustments may lead to a loss of efficiency and, ultimately, of control over the whole surgical process. Even if beds are available, strong fluctuations in bed occupancy during the week may cause further management problems, such as an unfair workload among nurses and doctors, as well as an uneven discharging process.

Actually, beds may even erroneously appear as a bottleneck, while their utilization over time only needs to be appropriately planned. In fact, as more hospitals embrace the *lean thinking* concept [13], there is a growing awareness about the importance of an integrated, pull-like management of the whole surgical pathway. In the literature, most of the approaches to MSSP and SCAP focus on OT operations, and do not consider the extended context in which the ORs are placed.

In this paper, we investigate surgical planning problems at the operational level from a broader perspective which considers surgical and post-surgical processes in an integrated way. We propose a new mathematical formulation for SCAP, and we show how it can be exploited as a management decision tool both in operational as well as strategic decision making.

The typical pathway for the post-surgical patient is to either admit the patient to an intensive care unit (ICU) if his/her health conditions are critical, or move him/her directly to a ward delivering the needed level of care. Specifically, we define three types of wards with different care levels: *Day Surgery* (or *Low-Intensity*, where the patient typically stays before being discharged home on the same day of the surgery), *Medium-Intensity* and *High-Intensity*.

Our model explicitly takes into account a number of issues which are encountered in clinical practice, but are often neglected, including:

- There are three different levels of *intensity of care*. These levels correspond to having different bedroom types, surgical cases and OR sessions.

- Beds are located into rooms, each room must contain patients of the same sex and having compatible levels of intensity of care.

- Surgical assignment takes into account the skill of the surgeons available on each day of the planning horizon.

In this paper we also illustrate the application of the model to the weekly planning of the OT in a medium-sized hospital in Tuscany. In particular, we explicitly take into account various tactical/strategic issues raised by the hospital management:

- *Week surgery vs. continuous surgery*. In most hospitals, even if the elective surgical activity takes place from Monday to Friday, post-surgical wards are always open, including the weekend. This is referred to as *continuous surgery*. A different organization aims at saving resources by closing down medium- and low-intensity wards during the weekends. This is referred to as *week surgery*. Clearly, in this case surgical planning must allow all patients operated during the week to be discharged within Friday night. Observe that, high-intensity wards are open during the weekend.

- *Dedicated bedrooms vs. shared bedrooms*. A given OT is shared by a number of surgical disciplines. The post-surgical bedrooms can be either partitioned among these disciplines (*dedicated* rooms) or, more flexibly, each bedroom can host patients from any disciplines (*shared* bedrooms). Observe that, the use shared bedrooms calls for an integrated planning system for the OT, since each surgical discipline should plan its surgeries considering the influence of the other disciplines in the shared bedrooms. On the other hand, when the bedrooms are dedicated the problem decompose nicely.

- *Bedroom sizing.* Given that each bedroom has a standard number of beds, the actual number of bedrooms to be employed by elective surgery may be decided on the basis of cost/benefit considerations. Surplus beds can be devoted to different uses, e.g., emergency or available to other surgical disciplines. Alternatively, the recognition of surplus bed capacity may allow management to increase surgical production, or, viceversa, to close bedrooms down.

Combining these policy issues in various ways, one obtains a number of different *scenarios*. To test our approach we considered a medium-size Italian hospital and we carried out tests on instances drawn from real data corresponding to different scenarios. The results show how this tool can help the hospital management to better investigate and quantify trade-offs arising when complex alternative scenarios have to be evaluated. Additionally, since the required computation times are limited, the tool can also be used to react to possible disruptions of the current surgical plan. Disruptions may be caused by unpredicted events such as longer/shorter than predicted LoS, severe emergencies, or changes in the surgeons' availability.

The paper is organized as follows. In Section 2 we briefly review related literature. In Section 3 the OR planning problem is described in detail, and in Section 4 the mathematical formulation is introduced. A case study concerning a medium-size Italian hospital, located in Prato (Tuscany) is illustrated and discussed in Section 5. Finally, in Section 6 some conclusions are drawn.

2. Literature review

In this section we briefly review the literature on papers considering the combined problems of OR planning and the effects of bed occupancy. We refer the readers to the recent reviews by [9, 14, 17] for a more comprehensive survey. In the literature several authors propose models for efficient OR usage ignoring resulting downstream bed utilization. When applied in practice, such results may cause congestions, unbalanced nurses workload [6] and bed occupancy, increased rejection rates [18] and surgeries cancellations. Few papers deal explicitly with OR planning and post-surgery bed occupancy, and the research on this topic can be divided into tactical-level and operational-level papers.

Developing a MSS is a difficult tactical process [14, 2] and, once adopted, the MSS is usually maintained for several months. This means that, the number of surgeries performed by each surgical discipline during each week varies, and bed occupancy over time is even more difficult to predict. Generally speaking, different MSSs cause different bed occupancy patterns in the wards, and some MSSs are preferable by management, being more balanced and having less fluctuations.

Vanberkel et al. [25] describe the iterative process followed to develop a new MSS at the NKI-AVL Amsterdam hospital. The iterative process consisted in adjusting the MSS by swapping OR blocks (discipline-day-room assignments), applying an a-posteriori statistical evaluation to compute the possible ward occupancy and let the management decide if the quality of the MSS was deemed sufficient. The resulting MSS helps to reduce fluctuations and peaks in the daily ward occupancy.

Blake and Carter [8] use a linear goal programming model to determine volume and mix of surgical cases. They consider patients' LoS and bed capacity as constraints, and they aim at optimizing financial performance for the hospital.

In [5] the authors consider the problem of generating a new MSS in such a way to minimize the total expected bed shortage. They propose different models to represent the total expected bed shortage in a tractable way. On these basis they develop several MIP based heuristics and a simulated annealing to solve randomly generated instances. In a follow up paper [7] they extend their model to consider also single surgeons, sessions of different durations and multiple wards. They develop a Decision Support System and show the results of its application to a Belgian hospital.

Santibáñez et al. [20] also introduce a mixed integer formulation to generate a MSS in which blocks are allocated to single surgical groups, with the objectives of minimizing the peak bed usage or throughput objectives to stabilize waiting lists. Their model works at single day planning level and consider multiple hospitals each with two types of beds, regular and special care beds. Different surgeries are aggregated into a single type and the problem is to find the number of surgeries assigned to each session.

Van Oostrum et al. [24] propose a two phase approach to construct a cyclic MSS minimizing the need OR capacity and peak demands of hospital beds. The overall problem is decomposed to generate operating rooms day schedules and then to assign operating rooms day schedules to the MSS in order to minimize the bed requirement. The two subproblems are solves by a column generation approach and by a mixed integer linear program, respectively.

In [12] the authors make use of a MIP model to produce a MSS and patient mix within each block with minimal bed requirements and use a Monte Carlo trace driven simulation to predict the daily bed occupancy. Additionally, based on the analysis of the results of their approach, they also provide a set of guidelines for the planners to manually produce surgical schedules.

Vissers et al. [26] introduced a mixed integer model taking into account ORs, medium and intensive care beds and nursing staff. The model is applied to a cardiothoracic surgery clinic and it produces the patient mix for each day of the planning horizon. as they minimize the weighted under- and over-utilization of each resource.

Recently, Van Essen et al. [23] considered a tactical level problem in which surgery types are scheduled and they assume a stochastic LoS. The non-linear objective function involving the convolution of discrete distributions aims at minimizing the number of required beds. Results based on random instances based on an hospital in the Netherlands show that the required beds can be reduced up to 20%.

All these papers address bed management as a tactical level problems, whereas the following papers focus on the operational level.

Augusto et al. [3] extend the OR planning problem to consider also two additional resources besides the ORs. Namely, they consider the presence of recovery beds and transporters. After surgery, the patient wakes up in a recovery bed, and if no recovery bed is available, the patient is kept in the OR, thus blocking it. Thereafter, the patient is moved to a ward through a transporter. The authors develop a Lagrangian relaxation technique to solve randomly generated realistic instances.

In [15], Guinet and Chaabane address an operating theatre composed of several ORs and one recovery room with several beds. Finite surgeon and equipment resources capacity is taken into account and the aim is to minimize costs incurred by the hospital. A heuristic for solving a general assignment problem with limited resource capacity and time-window constraints is proposed.

Jebali et al. [16] present a sequential two-phase approach for the scheduling of the operating rooms, which consists in first solving SCAP and then deciding the sequence of surgeries in each session. SCAP is solved by considering the total number of beds in the recovery rooms, whereas the sequencing problem is formulated as a two-stage hybrid flowshop with blocking constraints.

In all these operational-level planning models, beds in the wards are not modelled in detail. To the best of our knowledge, our approach is the first to address an integrated SCAP / bed management problem in which bedroom features and limitations are explicitly taken into account. In particular, we propose a detailed SCAP formulation in which OR sessions may have different lengths, and we consider restrictions concerning surgeon skills, surgeon availability and total workload, different bedroom sizes, different levels of intensity of care, and gender constraints on bedroom occupancy.

3. Problem description

In this paper we propose a new model for the surgical case assignment problem (SCAP) integrating operating theatre (OT) and the beds in the wards. Indeed, the beds are typically organized in rooms, and rooms are classified by intensity of care (IC). In this work we consider three levels of IC, namely *high*, *medium* and *low*. In particular, low care corresponds to day surgery patients.

On a given day, patients of different gender cannot occupy two beds in the same room. However, a room can host patients of different genders, on different days. Anyway, we assume that no patient changes his/her room during the recovery period.

In our framework, we assume that the MSS (i.e., assignment of surgical disciplines to OR sessions) is given and we allow *morning*, *afternoon* and *daily* sessions. All sessions of the same type have the same duration, which must not be exceeded by the total processing time of the surgeries allocated to that session.

We also explicitly consider the surgeons availability and skill. More specifically, the model assigns a surgeon to each surgical case, so that (*i*) for each surgical case, the surgeon is selected from a set of surgeons having an appropriate skill, and (*ii*) the surgical case must be scheduled in an OR session in which the selected surgeon is available. For each surgeon, the total weekly workload cannot exceed a given maximum value.

All relevant times for the decisions in this model are considered deterministic. While this may appear as a strong assumption, one should take into account the following:

- Surgical times can be obtained from historical data. To account for their uncertainty, one can decrease the duration of a OR session by a given slack time, in order to absorb, up to a certain extent, possible delays due to unpredicted events.
- Data concerning post-surgical length of stay can also be obtained from historical data. While a significant variability exists in the length of stay of patients undergoing urgent or emergency surgery, it is much less so for elective patients. This aspect is extremely important, but it is often overlooked by managers [4].

We next summarize the main "ingredients" of the problems we deal with, and for each of them we review in detail the attributes that are relevant to the decision making process.

(*i*) *Bedrooms.* A number of bedrooms is given, and for each of them the following parameters are specified:

  - *Intensity of care.* If a bedroom has *low* IC level, it can only host low-intensity (e.g., day surgery) patients; if it has *medium* IC level, it can host medium- or low-intensity patients; if it has *high* IC level, it can host any kind of patient.
  - *Capacity.* Number of beds in the room.

(*ii*) *Surgical cases.* For each surgical case in the waiting list the following information is given:

  - *Surgical discipline*
  - *Processing time.* This is the expected duration of the surgery (including setup times due to cleaning and OR preparation for the next surgery). This is also called surgical time. We assume it to be deterministic.
  - *Length of stay.* Expected duration of post-surgical patient stay (days), assumed to be deterministic. The stay starts on the day

in which the surgical case is performed. The length of stay includes the operation day and does not include the day when the patient is discharged.

- *Waiting time.* Days currently spent by the surgical case in the waiting list.
- *Priority class.* Surgeries are classified into three priority classes A, B or C (A being the most urgent), according to the regulatory essential assistance levels.
- *Intensity of care.* This specifies the level of care the patient will require after surgery, and can be *low*, *medium* or *high*.

(*iii*) *Surgeons.* A set of surgeons is given, and for each of them the following are specified:

- *Surgical discipline/skill.* Set of surgical cases the surgeon is able to perform, and can therefore be assigned to.
- *Weekly calendar.* The times in which surgeon is available, i.e., the set of OR sessions in which the surgeon can perform a surgical case.
- *Maximum weekly workload.* The maximum value of a surgeon's workload throughout the whole planning horizon, as the total surgical time assigned to him/her.

(*iv*) *OR Sessions.* For each OR session in the given master surgical schedule, the following information is given:

- *Intensity of care.* Surgical cases requiring high intensity of care must be performed in *high-intensity* OR sessions; the other surgical cases can be done in any OR session.
- *Surgical discipline.*
- *Session Type* A session can be either a *morning* or an *afternoon* session.
- *Day of the week.*
- *Operating room.*
- *Maximum duration.* Maximum time available for surgeries during that session.

(v) *Objective function.* The weekly plan of elective surgery should pursue the following objectives:

- Maximizing the utilization of ORs, without resorting to overtime as far as possible.
- Performing each case surgery as far as possible within the respective due date, i.e., performing the surgery before the waiting time exceeds the maximum allowed waiting time associated to the priority class of the case in the regulatory essential assistance levels.

In general, these two objectives may be partially conflicting. In fact, the former objective may lead to schedule surgeries that tightly fill OR sessions, but irrespective of the surgeries' due dates. On the other hand, scheduling surgeries based on their due dates only may lead to inefficient utilization of ORs. Hence, we define an objective function (to be maximized) that allows to account for both these aspects. Namely, we associate a *score* to each surgical case given by the product of the surgery processing time, the surgery waiting time and a coefficient depending on the priority class of the surgical case [1].

In the next section we introduce a mathematical formulation for the considered problem.

4. Mathematical model

In what follows, we first introduce the notation (Section 4.1) and then the mathematical formulation in Section 4.2.

4.1. Notation

In the optimization model that follows, we denote by $\mathcal{S}$ the set of surgical disciplines, $\mathcal{O}$ the set of surgical cases (patients) in the waiting list, $\mathcal{M}$ the set of OR sessions, $\mathcal{B}$ the set of bedrooms, Ω be the set of surgeons. Also, T denotes the length of the planning horizon (days). In our tests we assume a weekly planning horizon $T = 7$. We let $\mathcal{M}_s$ denote the set of OR sessions of surgical discipline s by the MSS, and by $\mathcal{M}_s^t \subseteq \mathcal{M}_s$ the set of OR sessions of discipline s that take place on day t. Also, we let $\mathcal{O}_F$ and $\mathcal{O}_M$ denote the set of female and male patients respectively.

For each $o \in \mathcal{O}$ we specify:

- the surgical discipline σ_o the surgery belongs to

- the set of surgeons Ω_o who are able to perform o
- the processing (surgical) time pt_o (minutes)
- the score w_o
- the length of stay ℓ_o (days)
- the patient gender, either F or M
- the required intensity of care $IC_o \in \{\text{High, Medium, Low}\}$,

For each session $m \in \mathcal{M}$ we specify:

- the operating day $t_m \in 1, \ldots, T$
- the surgical discipline S_m performed in the session
- the limit on session duration P_m^{max} (minutes)
- the intensity of care $IC^m \in \{\text{High, Normal}\}$

For each bedroom $b \in \mathcal{B}$ we specify:

- the number of beds N_b
- the intensity of care level $IC(b) \in \{\text{High, Medium, Low}\}$

Finally, for each surgeon $\omega \in \Omega$ we specify:

- the maximum weekly workload of the surgeon W_ω^{max} (minutes)
- the surgeon's skill, i.e., the set $\mathcal{O}_\omega$ of cases surgeon ω is able to perform
- the surgeon's calendar, i.e., the set $\mathcal{M}_\omega$ of OR sessions in which surgeon ω is available

4.2. Mathematical formulation

We next present the mathematical formulation of a model assuming continuous surgery and general-purpose bedrooms. We next illustrate how to modify the model to accommodate for the the week surgery and the preassigned bedrooms scenarios.

The decision variables of the models are the following.

- $x_{om} \in \{0,1\} \;\; \forall o \in \mathcal{O}, m \in \mathcal{M}_{\sigma(o)}$, where:

$$x_{om} = \begin{cases} 1 & \text{if surgical case } o \text{ is scheduled in OR session } m \\ 0 & \text{otherwise} \end{cases}$$

- $z_{obt} \in \{0,1\} \;\; \forall o \in \mathcal{O}, b \in \mathcal{B}, t \in 1, \dots, T$, where:

$$z_{obt} = \begin{cases} 1 & \text{if surgical case } o \text{ is performed in day } t \text{ and is allocated in bedroom } b \\ 0 & \text{otherwise} \end{cases}$$

- $\alpha_{o\omega} \in \{0,1\} \;\; \forall o \in \mathcal{O}, \omega \in \Omega$, where:

$$\alpha_{o\omega} = \begin{cases} 1 & \text{if surgical case } o \text{ is performed by surgeon } \omega \\ 0 & \text{otherwise} \end{cases}$$

We also introduce the auxiliary variables $y_{bt} \in \mathbb{N} \;\; b \in \mathcal{B}, t \in 1, \dots, T$ that indicate the number of patients that occupy bedroom b for the *entire* day t, i.e., which entered the room before t, and that still occupy the room after t. Note that y_{bt} does not include the patients discharged on t. In other words, we are assuming that the bed of patients discharged on day t is already available for newly entered patients on the same day. Notice that variables y_{bt} depend on the binary variables defined above.

The mathematical formulation is:

$$\max \sum_{o\in\mathcal{O}} \sum_{m\in\mathcal{M}_{\sigma(o)}} w_o \cdot x_{om} \tag{1}$$

$$\sum_{m\in\mathcal{M}_{\sigma(o)}} x_{om} \leq 1 \quad \forall o \in \mathcal{O} \tag{2}$$

$$\sum_{o\in\mathcal{O}_\omega} pt_o \alpha_{o\omega} \leq W_\omega^{max} \quad \forall \omega \in \Omega \tag{3}$$

$$\sum_{o\in\mathcal{O}} pt_o x_{om} \leq P_m^{max} \quad \forall m \in \mathcal{M} \tag{4}$$

$$\sum_{m\in\mathcal{M}_{\sigma(o)}} x_{om} = \sum_{\omega\in\Omega_o} \alpha_{o\omega} \quad \forall o \in \mathcal{O} \tag{5}$$

$$(1-\alpha_{o\omega}) \geq x_{om} \quad \forall o \in \mathcal{O}, \omega \in \Omega_o, m \notin \mathcal{M}_\omega \tag{6}$$

$$\sum_{o:IC_o=High} x_{om} = 0 \quad \forall \ m \in \mathcal{M} : IC^m = \text{Normal} \tag{7}$$

$$\sum_{m\in\mathcal{M}^t_{\sigma(o)}} x_{om} = \sum_{b\in\mathcal{B}} z_{obt} \ \ \forall o \in \mathcal{O}, t = 1 \ldots T \tag{8}$$

$$\sum_{o:IC_o=High} z_{obt} = 0 \quad \forall \ b \in \mathcal{B} : IC(b) = \text{Medium or Low}, t = 1 \ldots T \tag{9}$$

$$\sum_{o:IC_o=Medium} z_{obt} = 0 \quad \forall \ b \in \mathcal{B} : IC(b) = \text{Low}, t = 1 \ldots T \tag{10}$$

$$\sum_{i\in\mathcal{O}_F} \sum_{\tau=t}^{t+\ell_i-1} z_{ib\tau} \leq M \cdot (1 - z_{obt}) \quad \forall o \in \mathcal{O}_M, b \in \mathcal{B}, t = 1 \ldots T \tag{11}$$

$$\sum_{i\in\mathcal{O}_M} \sum_{\tau=t}^{t+\ell_i-1} z_{ib\tau} \leq M \cdot (1 - z_{obt}) \quad \forall o \in \mathcal{O}_F, b \in \mathcal{B}, t = 1 \ldots T \tag{12}$$

$$\sum_{o\in\mathcal{O}} z_{obt} \leq N_b - y_{tb} \qquad \forall b \in \mathcal{B}, t = 1 \ldots T \tag{13}$$

$$y_{t+1,b} + \sum_{o\in\mathcal{O}} z_{ob(t+1-\ell_o)} = y_{tb} + \sum_{o\in\mathcal{O}} z_{obt} \quad \forall t = 1 \ldots T-1, b \in \mathcal{B} \tag{14}$$

Constraint (2) states that each surgery case can be performed at most once. Constraints (3) and (4) enforce the limits on the maximum workload of each surgeon and the duration of an OR session respectively. Constraints

(5) imply that each selected surgical case is assigned to an adequate surgeon. Constraints (6) establish that, if a surgical case o is assigned to a surgeon ω, o must be scheduled in a session in which surgeon ω is available. Constraints (7) prevent a high-intensity surgical case to be assigned to a normal-intensity OR session. Constraints (8) imply that if a surgical case is scheduled on day t, the patient must start his/her post-surgical stay in some bedroom on day t. Constraints (9) and (10) prevent a patient requiring a certain intensity of care to be allocated in a bedroom having lower intensity of care. Constraints (11) impose that, if a male patient o is operated on a day t and is assigned to a room b, that room cannot host female patients during the stay of patient o. Symmetrically, constraints (12) impose the same for female patients. In these constraints, M is a sufficiently large integer (e.g., it suffices to set $M > TN_b$). Constraints (13) impose that patients entering room b on day t cannot exceed the number of currently free beds in that room. Constraints (14) define the inventory variables y_{tb}. In fact, on day $t+1$ there are the patients present on day t, plus new arrivals on t, minus the patients who are discharged on $t+1$.

The Model (1)–(14) can be easily extended to consider two variants of the problem. Namely, we consider the week surgery and the preassigned bedrooms scenarios.

When considering the preassigned bedrooms we have to add the following notation. We let $\mathcal{B}_s$ denote the set of bedrooms of the surgical discipline s and by $\mathcal{O}_s$ the set of surgical cases of the surgical discipline s. Then Equation (15) represents the bedrooms preassignment constraint.

$$\sum_{o \notin \mathcal{O}_s} \sum_{t=1}^{T} z_{obt} = 0 \quad \forall s \in \mathcal{S}, b \in \mathcal{B}_s \tag{15}$$

Whereas the week surgery case is modelled by adding Constraint (16) to the Model (1)–(14). In fact, it guarantee that no bed with medium- or low-intensity patients are occupied during the weekend.

$$\sum_{o \in \mathcal{O}:\{t+\ell_o \geq T-1\}} z_{obt} = 0 \quad \forall b \in \mathcal{B} : IC_b \notin \{High\}, t \in \{1, \ldots, T\} \tag{16}$$

Finally, when considering a scenario with both the week surgery and the preassigned bedrooms, then Constraints (15) and (16) are added to Model (1)–(14).

Table 1: MSS of the Prato Hospital.

	Monday		Tuesday		Wednesday		Thursday		Friday	
$OR\,1$	ENT		GS		ENT		URO		ENT	
$OR\,2$	URO		GS	GS	GS		GS		GS	---
$OR\,3$	GYN		GYN		GS	GS	GS		GYN	---
$OR\,4$	GS	---	ENT	---	ENT	---	GS	---	GS	---

5. Case study and computational results

5.1. Case study

As a case study we consider the multidisciplinary operating theatre of the hospital Misericordia e Dolce located in Prato (Tuscany, Italy). Such OT is the critical theatre of the hospital both in terms of size and complexity. It consists of four ORs and serves four surgical disciplines: General surgery (GS), otolaryngology (referred to as ear-nose-throat, ENT), gynaecology (GYN) and urology (URO). Each of the four disciplines has its own waiting list (I_s) and the Master Surgical Schedule of the OT is shown in Table 1.

There are three types of OR sessions, each with a different duration:

- morning session (420 minutes),
- afternoon session (300 minutes),
- full-day session (720 minutes).

The OR management plans surgical activities of each morning session so that a nominal length of 390 minutes is not exceeded, hence keeping a 30-minute buffer to absorb possible uncertainties in surgeries' duration. Similarly, the length of afternoon and full-day sessions is assumed to be 270 and 660 minutes respectively.

The operating rooms in the theatre are not identical. In particular, high-intensity surgeries ($IC_o = High$) cannot be performed in OR4 (constraint (7)), whereas all the remaining operating rooms are equipped to perform high-intensity surgeries.

The ward is composed of 21 bedrooms ($|\mathcal{B}| = 21$) shared by the surgical disciplines. The rooms are divided into three groups according to the required level of intensity of care. The capacity of the bedrooms ranges from 1 to 5 beds. Let B_i be the number of bedrooms having i beds. Table 2 shows the number of rooms for each capacity value and for each level of intensity of care (High, Medium, Low). A given bedroom can host only patients of the same gender. Note that in general, patients of different disciplines may be hosted in the same bedroom.

Table 2: Details on the bedroom capacities divided by intensity of care (case $|\mathcal{B}| = 21$).

Intensity of Care	B_1	B_2	B_3	B_4	B_5	Total Beds
High	0	1	3	4	0	27
Medium	0	0	1	0	4	23
Low	8	0	0	0	0	8

There are 51 available surgeons. In particular, there are 25 surgeons for General Surgery ($|\Omega_{\text{GS}}| = 25$), 12 ear-nose-throat surgeons ($|\Omega_{\text{ENT}}| = 12$), 8 gynecology surgeons ($|\Omega_{\text{GYN}}| = 8$) and 6 urology surgeons ($|\Omega_{\text{URO}}| = 6$). Each surgeon can work at most 1500 minutes (25 hours) per week.

5.2. Experimental design and setting

We next describe how the benchmark instances have been generated. In our tests, we considered different scenarios on the basis of four different issues: the number of available bedrooms (3 cases), the MSS (5 cases), the bedroom sharing policy (2 cases), the weekend policy (2 cases). Overall, we considered 60 scenarios.

For the number of available bedrooms, we considered $|\mathcal{B}| = \{15, 18, 21\}$. There are currently 21 bedrooms (Table 2). The details of the case with 15 and 18 bedrooms are reported in Tables 3 and 4, respectively.

Table 3: Details on the bedroom sizes divided by care level (case $|\mathcal{B}| = 15$).

Intensity of Care	B_1	B_2	B_3	B_4	B_5	Total Beds
High	0	1	1	2	0	13
Medium	0	0	1	0	2	13
Low	8	0	0	0	0	8

Table 4: Details on the bedroom sizes divided by care level (case $|\mathcal{B}| = 18$).

Intensity of Care	B_1	B_2	B_3	B_4	B_5	Total Beds
HIGH	0	1	3	3	0	23
MEDIUM	0	0	0	0	3	15
LOW	8	0	0	0	0	8

Table 5: MSS of the Prato Hospital (case $|M| = 26$).

	Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
$OR\,1$	ENT		GS		ENT		URO		ENT		GS	---
$OR\,2$	URO		GS	GS	GS		GS		GS	---	ENT	---
$OR\,3$	GYN		GYN		GS	GS	GS		GYN	---	GS	---
$OR\,4$	GS	---	ENT	---	ENT	---	GS	---	GS	---	ENT	---

The second considered issue is the MSS. We considered 5 different MSSs. The first MSS is the as-is situation, in which $|M| = 22$ (Table 1). The others are obtained from the as-is situation by adding 1, 2, 3 and 4 additional Saturday morning sessions respectively. The case with $|M| = 26$, in which all 4 sessions have been added, is reported in Table 5.

Regarding the bedroom sharing policy, we indicate it by BS, and we consider two cases $BS = \{shared, dedic\}$. The as-is case is $BS = shared$ (Table 2), i.e., the beds in each bedroom can be used by any surgical discipline. In the case $BS = dedic$, the bedrooms are pre-assigned to specific disciplines. In this case, the bedroom preassignment is reported in Table 6.

The weekend policy can be either $WP = week$ or $WP = cont$, standing respectively for week surgery and continuous surgery.

Hence, a scenario can be described by the tuple $S(|\mathcal{B}|, |M|, BS, WP)$.

In order to run our tests, we sampled 10 weeks in the time interval ranging from January 2010 through June 2013, employing the corresponding waiting lists. In a given waiting list $\mathcal{O}$, we denote by $\mathcal{O}_A$, $\mathcal{O}_B$ and $\mathcal{O}_C$ the subsets of surgeries having priority A, B and C respectively. Moreover, we assumed the presence in the ward of a number of patients from the previous week. The number of such patients as well as their residual length of stay have been estimated from historical data. For each of the 10 instances, the cardinality of each subset is reported in Table 7.

Each of the 10 instances is run for each scenario, by solving the ap-

Table 6: Details on room sizes divided by care level pre-assigned to each surgical discipline.

	$\|\mathcal{B}\|$=15		$\|\mathcal{B}\|$=18		$\|\mathcal{B}\|$=21	
Surgical discipline	Number of rooms	Number of beds	Number of rooms	Number of beds	Number of rooms	Number of beds
GS	6	17	8	23	11	30
ENT	5	7	6	10	6	12
GYN	2	5	2	7	2	9
URO	2	5	2	6	2	7

Table 7: Number of surgeries in the waiting lists grouped by priority

Instance	$\|\mathcal{O}_A\|$	$\|\mathcal{O}_B\|$	$\|\mathcal{O}_C\|$	$\|\mathcal{O}\|$
1	111	102	259	472
2	169	146	182	497
3	160	93	181	434
4	179	126	182	487
5	208	115	153	476
6	228	114	263	505
7	244	108	148	500
8	250	93	162	505
9	248	117	143	508
10	245	121	134	500

propriate mathematical model. Such model depends on BS and WP, as summarized in Table 8. Hence, a total of 600 runs is obtained.

Table 8: Mathematical models for various scenarios.

(BS, WP)	model
$shared, cont$	(1)–(14)
$dedic, cont$	(1)–(14) and (15)
$shared, week$	(1)–(14) and (16)
$dedic, week$	(1)–(14), (15) and (16)

5.3. Numerical results

Tests have been performed on a 3.2 GHz Intel Core i3 processor with 4 GB of RAM, using OPL Studio 6.1 and the CPLEX 11.2 MILP solver. In each of the 600 runs, the maximum computation time has been set to 900 seconds. In all runs, the final optimality gap was always smaller than 1%.

Tables 9–11 report on the results with $|\mathcal{B}| = 15, 18$ and 21, respectively. All figures are average values over 10 instances. We report the following performance indices:

- *Number Sched* is the total number of surgeries planned during the week
- *Number Sched A,B,C* are the breakdown of planned surgeries by priority class
- *% Occ. OR* is the occupancy rate of the OR sessions, i.e., the ratio between the total surgical time planned in the week and the total duration of all OR sessions (not including the buffer time)
- *% Occ. Beds* is the occupancy rate of the beds in the ward, i.e., the ratio between the total number of inpatient days and bed-days available in the week. Notice that the latter figure is obtained by considering that all bed are available for 7 days per week for each scenario ($WP = week$ and $WP = cont$)
- *Discharged next week* indicates the number of patients which are not discharged during the current week.

Table 9: Results for the scenarios with $|B| = 15$

Scenario	Number Sched	Number Sched A	Number Sched B	Number Sched C	% Occ. OR	% Occ. Beds	Discharged next week
$S(15, 22, shared, cont)$	94.70	66.40	8.80	19.20	93.12	76.68	7.00
$S(15, 22, dedic, cont)$	71.70	55.70	6.10	9.80	89.96	57.69	5.10
$S(15, 22, shared, week)$	94.20	65.10	9.60	19.20	91.10	70.80	6.10
$S(15, 22, dedic, week)$	71.60	54.20	7.40	9.90	83.69	53.45	4.40
$S(15, 23, shared, cont)$	98.80	71.80	8.70	18.00	93.00	78.51	9.20
$S(15, 23, dedic, cont)$	78.80	63.28	6.35	9.06	82.50	64.12	7.70
$S(15, 23, shared, week)$	96.70	74.60	6.50	15.20	87.93	74.55	7.20
$S(15, 23, dedic, week)$	77.50	63.60	5.20	8.60	81.58	61.40	7.30
$S(15, 24, shared, cont)$	101.30	72.90	8.00	20.10	87.23	77.18	9.40
$S(15, 24, dedic, cont)$	86.30	66.00	7.60	12.60	81.52	62.86	8.80
$S(15, 24, shared, week)$	103.00	74.20	7.70	20.80	88.28	73.32	9.40
$S(15, 24, dedic, week)$	85.40	62.80	8.90	13.60	80.70	57.86	7.30
$S(15, 25, shared, cont)$	109.00	78.20	9.40	21.10	91.84	81.93	8.60
$S(15, 25, dedic, cont)$	86.40	67.20	7.40	11.70	81.64	65.34	6.60
$S(15, 25, shared, week)$	107.20	77.70	8.90	20.30	87.01	74.94	6.10
$S(15, 25, dedic, week)$	85.50	65.70	7.80	11.90	80.08	61.63	4.80
$S(15, 26, shared, cont)$	111.90	83.10	9.50	19.00	92.65	83.53	9.30
$S(15, 26, dedic, cont)$	88.10	70.30	6.70	11.00	82.39	67.02	7.50
$S(15, 26, shared, week)$	110.50	83.00	9.10	18.20	90.79	75.38	9.00
$S(15, 26, dedic, week)$	86.80	66.80	8.30	11.60	81.31	62.13	6.50

Table 10: Results for the scenarios with $|B| = 18$

Scenario	Number Sched	Number Sched A	Number Sched B	Number Sched C	% Occ. OR	% Occ. Beds	Discharged next week
$S(18, 22, shared, cont)$	98.40	67.80	9.90	20.40	96.00	65.25	10.30
$S(18, 22, dedic, cont)$	78.20	58.60	7.80	11.50	84.56	48.76	9.10
$S(18, 22, shared, week)$	97.60	67.40	9.10	20.80	95.40	64.72	9.10
$S(18, 22, dedic, week)$	77.40	57.90	7.80	11.40	84.46	46.33	8.00
$S(18, 23, shared, cont)$	100.50	74.80	6.40	18.80	95.34	65.25	6.70
$S(18, 23, dedic, cont)$	85.50	69.20	5.90	10.20	84.82	47.11	5.20
$S(18, 23, shared, week)$	100.50	75.20	7.30	17.60	92.97	64.58	7.30
$S(18, 23, dedic, week)$	84.90	68.80	5.30	10.60	82.23	46.97	4.70
$S(18, 24, shared, cont)$	106.10	74.50	8.80	22.50	89.56	66.54	10.70
$S(18, 24, dedic, cont)$	92.60	67.40	9.00	16.10	84.55	51.93	7.90
$S(18, 24, shared, week)$	105.50	73.60	8.50	23.10	88.58	66.38	10.90
$S(18, 24, dedic, week)$	90.00	66.80	7.70	15.40	83.34	50.37	9.30
$S(18, 25, shared, cont)$	113.70	79.70	10.10	23.60	94.45	71.40	8.10
$S(18, 25, dedic, cont)$	94.00	72.30	8.00	13.60	84.88	54.41	6.60
$S(18, 25, shared, week)$	113.30	80.00	10.30	22.70	93.82	70.47	8.30
$S(18, 25, dedic, week)$	92.10	70.80	7.90	13.30	84.78	52.27	5.50
$S(18, 26, shared, cont)$	115.90	83.80	9.30	22.30	93.47	71.00	9.90
$S(18, 26, dedic, cont)$	95.10	73.70	8.70	12.50	85.14	54.84	8.40
$S(18, 26, shared, week)$	114.50	82.60	9.50	21.80	93.16	69.53	9.80
$S(18, 26, dedic, week)$	93.40	73.30	7.60	12.30	85.91	53.04	7.60

Table 11: Results for the scenarios with $|B| = 21$

Scenario	Number Sched	Number Sched A	Number Sched B	Number Sched C	% Occ. OR	% Occ. Beds	Discharged next week
$S(21, 22, shared, cont)$	99.30	67.50	10.50	21.00	96.06	49.26	6.90
$S(21, 22, dedic, cont)$	86.10	54.10	9.30	22.10	85.32	45.67	7.20
$S(21, 22, shared, week)$	98.30	67.60	9.60	20.80	95.68	48.10	6.70
$S(21, 22, dedic, week)$	85.60	58.90	9.30	17.10	85.22	44.83	6.40
$S(21, 23, shared, cont)$	100.60	74.50	7.30	18.30	95.69	52.52	9.20
$S(21, 23, dedic, cont)$	92.30	67.80	6.70	17.50	92.82	50.94	9.80
$S(21, 23, shared, week)$	100.50	76.30	6.50	17.10	94.66	51.21	8.50
$S(21, 23, dedic, week)$	91.30	67.50	6.80	16.70	92.24	48.72	10.20
$S(21, 24, shared, cont)$	107.10	75.60	8.50	22.70	90.04	54.38	8.20
$S(21, 24, dedic, cont)$	96.60	66.40	9.40	20.60	86.55	50.76	7.10
$S(21, 24, shared, week)$	107.10	74.20	9.40	23.10	89.92	53.50	8.20
$S(21, 24, dedic, week)$	96.30	66.40	8.70	21.00	85.85	50.32	7.90
$S(21, 25, shared, cont)$	114.50	80.50	10.60	23.10	96.06	55.86	9.00
$S(21, 25, dedic, cont)$	101.70	73.80	8.60	19.10	89.87	50.94	8.30
$S(21, 25, shared, week)$	113.70	78.90	10.60	23.90	94.94	54.68	9.30
$S(21, 25, dedic, week)$	101.00	73.50	8.50	18.80	89.72	50.12	8.70
$S(21, 26, shared, cont)$	118.50	84.70	9.90	23.40	94.71	50.76	10.60
$S(21, 26, dedic, cont)$	103.20	76.60	8.50	17.80	91.36	46.75	9.70
$S(21, 26, shared, week)$	117.60	84.40	9.90	22.80	94.60	50.10	9.80
$S(21, 26, dedic, week)$	103.00	76.20	9.10	17.40	90.27	46.70	9.40

From the results in Tables 9–11 a few comments are in order:

- The best performance in terms of OR occupancy is obtained when beds are shared among surgical disciplines ($BS = shared$). Correspondingly, also bed occupancy is higher. In fact, on the average the scenarios with $BS = dedic$ yield an OR occupancy which is 7% smaller, while bed occupancy is 12% smaller.

- The scenarios with $WP = week$ and $WP = cont$ yield very similar results. In fact, scenarios with $WP = cont$ have a slightly higher OR and bed occupancy rates but a larger number of patients during the weekend.

- Increasing the number of available OR sessions ($|M| > 22$), both surgical activity and bed occupancy increase. This shows that the beds are not an actual bottleneck and that the productivity can still be increased without adding other beds.

- Not surprisingly, there is a substantial increase in planned surgeries and OR utilization when moving from 15 to 18 rooms. Also, the number of patients remaining in the hospital during the weekend (as shown in the last column of Tables 9–11) is smaller when $|\mathcal{B}| = 15$. This suggests that the management should consider the scenario with 18 rooms as the best from the service level point of view, whereas having only 15 rooms allow to reduce the work during the weekend while having a good service level. However, the marginal increase when moving from 18 to 21 bedrooms is much smaller. More in details, there are no differences between using 18 and 21 bedrooms when beds are shared among surgical disciplines ($BS = shared$). When bedrooms are preassigned ($BS = dedic$) increasing the number of available beds yield to an improvement in the OR occupancy of about 4%. This means that, with 18 bedrooms, beds are a bottleneck only in the case of *dedic* policy, and not when bedrooms are shared among disciplines.

5.4. Managerial implications

In this section we want to show how the consideration of beds may affect planning decisions. In fact, not considering beds may induce the misleading impression that the ward is undersized even when it is simply poorly planned. To show this, we compare the results of two models for a specific test case: *(i)* an *integrated model* including bed levelling constraints (Model (1)–(16)) and *(ii)* a *simple model* disregarding the presence of bedrooms, i.e., in which all the constraints related to bedrooms are removed (Model (1)–(7)). The scenario considered in the test case is $S(15, 22, shared, cont)$, i.e., 15 shared bedrooms with the continuous weekend policy and no additional Saturday morning OR openings. In Figure 1 we show the comparison between the two approaches in terms of bed occupancy level.

In the simple model (model *(ii)*), bed occupancy has a peak from Tuesday to Thursday. The presence of these peaks causes problems to management, including an increased workload of nurses and doctors, early patient discharges, movement of patients to other wards, and even rescheduling of some planned surgery due to bed unavailability. On the other hand, when the beds are accounted for (model *(i)*), bed occupancy is more balanced throughout the week, without peaks that exceed bed availability. However, the consideration of bedroom constraints reduces the output from the OT. In fact, model *(ii)* plans 105 surgeries and the OR occupancy rate is 98.21%. On the other hand, when model *(i)* is adopted, the number of surgeries is 99 and the operating room occupancy rate drops to 95.76%. Observe that, the 105 weekly surgeries can be attained only at the price of peaks exceeding the available

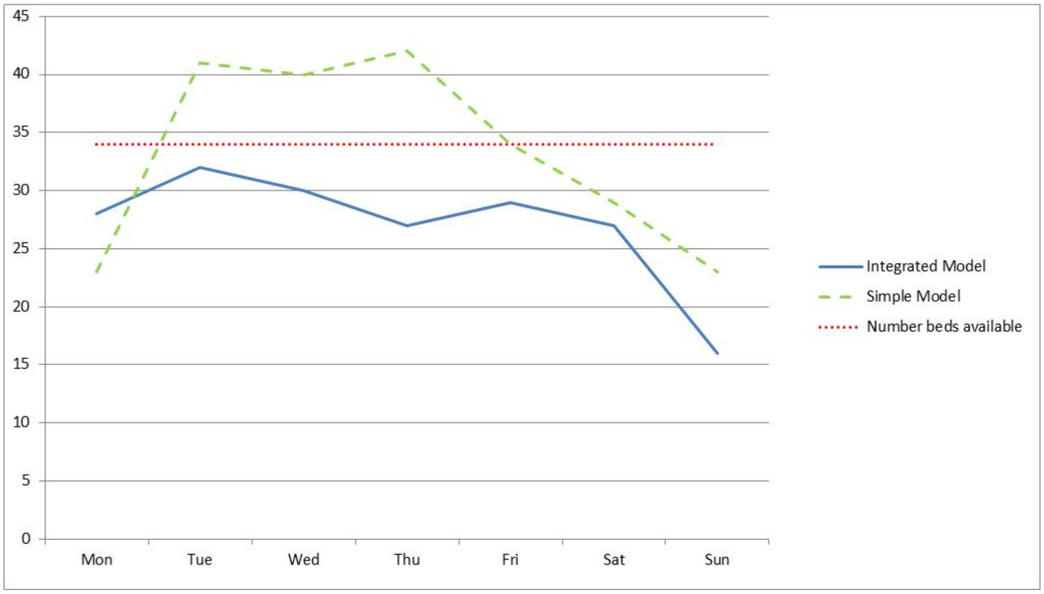

Figure 1: Bed occupancy during the week.

beds and hence, as previously argued, an expectedly lower service level for the patients.

The presence of peaks in weekly bed occupancy may induce managers to perceive beds as a bottleneck of the surgical path. This example shows that this may be only partially true, since careful planning (accounting for bed constraints) may smooth out peaks paying a small price in terms of surgical output, avoiding additional problems caused by bed unavailability.

6. Conclusions and future research

The main purpose of our study was to integrate in a single manageable model the planning of an operating theatre and the subsequent wards in such a way that the bed occupancy is levelled and without peaks. Moreover we assessed different possible configurations or management policies that can be adopted by the hospital management once a good integration between OR and ward is available. In fact we addressed (*i*) the case of adding OR sessions on Saturday, (*ii*) the possibility to close the medium and low intensity rooms during the weekend, thus reducing the work on the weekend, and (*iii*) the influence of sharing the beds among different surgical disciplines vs. the case of preassigning rooms to a discipline. Having such support tool allow the hospital management to quickly evaluate benefits and gains from different options, to compare them with the organizational costs related to each option and allow for quick adjustment to react to changed external environment.

The results of the experiments, based on an Italian hospital, suggest that having a model with detailed bed levelling can shave the peaks in bed occupancy while maintaining high throughput rate of the ORs. Moreover, each evaluated configuration yields to some benefit that can be quantified and help the hospital management to take informed decisions.

Future research may address possible refinements and improvements of the models presented, such as:

- including the presence of ICU and recovery rooms beds and ward beds
- including uncertainties (e.g., in surgical case durations and length of stay)
- adopt a multiobjective optimization approach, to explicitly account for more than one objective, besides what captured by the current objective function.

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[1] Agnetis, A., Coppi, A., Corsini, M., Dellino, G., Meloni, C., Pranzo, M., *Long term evaluation of operating theater planning policies*, Operations Research for Health Care, 1 (4), 95–104 (2012).

[2] Agnetis, A., Coppi, A., Corsini, M., Dellino, G., Meloni, C., Pranzo, M., *A decomposition approach for the combined master surgical schedule and surgical case assignment problems*, Health Care Management Science, to appear, DOI 10.1007/s10729-013-9244-0

[3] Augusto, V., Xie, X., Perdomo, V., *Operating theatre scheduling with patient recovery in both operating rooms and recovery beds*, Computers and Industrial Engineering, 58, 231–238 (2010).

[4] Baker, M., Taylor, I., Mitchell, A., (2009), *Making hospitals work*, Lean Enterprise Academy, Goodrich, UK.

[5] Beliën J., Demeulemeester E., (2007) *Building cyclic master surgery schedules with leveled resulting bed occupancy*, European Journal of Operational Research, 176, 1185-1204.

[6] Beliën J., Demeulemeester E., (2008) *A branch-and-price approach for integrating nurse and surgery scheduling*, European Journal of Operational Research, 189 (3), 652-668.

[7] Beliën J., Demeulemeester E., Cardoen B., (2009) *A decision support system for cyclic master surgery scheduling with multiple objectives*, Journal of Scheduling, 12, 147-161.

[8] Blake J.T., Carter M.W., (2002) *A goal programming approach to strategic resource allocation in acute care hospitals*, European Journal of Operational Research, 140, 541-561.

[9] Cardoen B., Demeulemeester E., Beliën J., *Operating Room planning and scheduling: a literature review*, European Journal of Operational Research, 201, 921–932 (2010).

[10] Cerda E., De Pablos L., Rodriguezuria M.V., (2006) *Waiting Lists for Surgery*. In: R.W. Hall (ed.), Patient Flow: Reducing Delay In Healthcare Delivery, International Series In Operations Research & Management Science, Springer, 151–188.

[11] Chaabane S., Meskens N., Guinet A., Laurent M., *Comparison of two methods of operating theatre planning: Application in Belgian Hospital*, Journal of Systems Science and Systems Engineering, 17 (2), 171–186 (2008).

[12] Chow V.S., Puterman M.L., Salehirad N., Huang W., Atkins D. (2011) *Reducing Surgical Ward Congestion Through Improved Surgical Scheduling and Uncapacitated Simulation*, Production and Operations Management, 20, 418–430.

[13] Collar R.M., Shuman A.G., Feiner S., McGonegal A.K., Heidel N., Duck M., McLean S.A., Billi J.E., Healy D.W., Bradford C.R., (2012) *Lean Management in Academic Surgery*, Journal of the American College of Surgeons, 214 (6), 928-936.

[14] Guerriero F., Guido R., (2011) *Operational research in the management of the operating theatre: a survey*, Health Care Management Science, 14 (1), 89–114.

[15] Guinet A., Chaabane S., *Operating theatre planning*, International Journal of Production Economics, 85, 69–81 (2003).

[16] Jebali A., Alouane A.B.H., Ladet P., (2006) *Operating rooms scheduling*, International Journal of Production Economics, 99 (1-2), 52–62.

[17] May J.H., Spangler W.E., Strum D.P., Vargas L.G., (2011) *The Surgical Scheduling Problem: Current Research and Future Opportunities*, Production and Operations Management, 20, 392–405.

[18] McManus M.L., Long M.C., Cooper A., Litvak E., (2004) *Queuing theory accurately models the need for critical care resources*, Anesthesiology, 100, 1271–1276.

[19] Roland B., Di Martinelli C., Riane F., Pochet Y., (2010) *Scheduling an operating theatre under human resource constraints*, Computer and Industrial Engineering, 58 (2), 212-220.

[20] Santibáñez P., Begen M., Atkins D., (2007) *Surgical block scheduling in a system of hospitals: an application to resource and wait list management in a British Columbia health authority*, Health Care Management Science, 10, 269–282.

[21] Sobolev B., Levy A., Kuramoto L., (2006) *Access to Surgery and Medical Consequences of Delays*. In: R.W. Hall (ed.), Patient Flow: Reducing Delay In Healthcare Delivery, International Series In Operations Research & Management Science, Springer, 79–100.

[22] Testi A., Tanfani E., Torre G.C., (2007) *A three phase approach for operating theatre schedules*, Health Care Management Science, 10, 163–172.

[23] van Essen J.T., Bosch J.M., Hans E.W., van Houdenhoven M., Hurink, J.L. (2013) *Reducing the number of required beds by rearranging the OR-schedule*, OR Spectrum, to appear.

[24] van Oostrum J.M., van Houdenhoven M., Hurink J.L., Hans E.W., Wullink G., Kazemier G., (2008) *A master surgical scheduling approach for cyclic scheduling in operating room departments*, OR Spectrum, 30, 355–374.

[25] Vanberkel P.T., Boucherie R.J., Hans E.W., Hurink J.L., van Lent W.A.M., van Harten W.H., (2011) *Accounting for Inpatient Wards When Developing Master Surgical Schedulesg*, Anesthesia & Analgesia, 112, 1472–1479.

[26] Vissers J.M.H., Adan I.J.B.F., Bekkers J.A., (2005) *Patient mix optimization in tactical cardiothoracic surgery planning: a case study*, IMA Journal of Management Mathematics, 16 (3), 281–304.