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Decomposition Approaches for the combined Master Surgical Schedule and Surgical Case Assignment Problems

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Abstract This paper addresses decomposition methods for Operating Room (OR) planning and scheduling. More specifically, we first focus on determining the Master Surgical Schedule (MSS) on a weekly basis, by assigning the different surgical disciplines to the available sessions. Next, we allocate surgeries to each session, focusing on elective patients only. Patients are selected from the waiting lists according to several parameters, including surgery duration, waiting time and priority class of the operations. We apply the proposed models to the Operating Theater (OT) of a public hospital in Empoli, Italy. We set up a campaign of computational experiments aimed at investigating the performance of alternative policies for the management of the MSS and the waiting lists over a one-year time horizon. The proposed decomposition approach is effective in identifying satisfactory solutions with significant savings in computation time.

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1 Introduction

The operating theater (OT), consisting of several *operating rooms* (ORs), is one of the most critical resources in a hospital because it has a strong impact on the quality of health service and represents one of the main sources of costs (surgical teams, equipment etc.). Given the patients' waiting list and various information on OT characteristics and status, OT planning problems consist in deciding the schedule of surgeries in a given time horizon, with the aim of optimizing several performance measures such as operating room utilization, throughput, surgeons' overtime, lateness etc.

Surgical cases are carried out in *OR sessions*. An OR session is an uninterrupted time block (typically, half day or a full day). In the management policy usually referred to as *block scheduling* [6], each OR session is devoted to a specific surgical discipline. This organizational solution is often preferred, since having the same type of surgeries performed in a given room during a given time span typically simplifies the physical handling of equipment and/or materials. A more flexible solution is the *open scheduling* policy [4], in which no pre-specified session-to-discipline assignment exists, and therefore two cases corresponding to different disciplines can be scheduled in the same OR session. This paper focuses on the block scheduling scenario. Thus, surgical planning in operating theaters can be seen as involving three distinct decision steps [16]:

(i) deciding the surgical discipline that will be performed in each OR session (e.g., the OR session "Tuesday morning–Room 2" is assigned to otolaryngology etc)
(ii) selecting elective surgeries to be performed in each OR session
(iii) sequencing surgeries within each OR session.

Problem (i) is often referred to as the Master Surgical Scheduling Problem (MSSP), and returns the *Master Surgical Schedule* (MSS) [3]. Problem (ii) determines the *Surgical Case Assignment* (SCA), and is therefore denoted as Surgical Case Assignment Problem (SCAP). Problem (iii) outputs the detailed calendar of elective surgeries for each session.

Literature on all three above decision levels is wide and growing, and it has thoroughly been reviewed by several researchers (e.g., [3, 8, 17, 20]). The three above decision problems have been addressed by a multiplicity of approaches. Research focused on either all three levels concurrently [16], two of them [12, 7, 19], or even single problems [18, 21, 5]. Some approaches have been designed to fit specific issues that may or may not be present in various real-life settings. For instance, Roland et al. [14] propose a model that incorporates requirements related to personnel availability and preferences, Augusto et al. [2] take into account bed occupancy in the wards, Guinet and Chaabane [9] include various hospitalization and overtime cost figures. Denton et al. [5] consider uncertain durations of surgeries while in [10] uncertainties in the surgeries demands levels are considered etc.

In this paper we focus on efficiently solving MSSP and SCAP, i.e., the first two steps of the decision hierarchy. Our assumptions are similar to those in the papers by Testi et al. [17, 15] and Agnetis et al. [1], who propose an ILP model for MSSP and SCAP, on the basis of the current state of the waiting list. Their approach consists in concurrently defining the MSS and the list of surgical cases to be performed during each OR session of the planning horizon.

The main contribution of this paper is to assess the quality of an efficient *decomposition approach* to MSSP and SCAP. This approach is compared with the solution of an integrated ILP model which concurrently solves the two problems [1]. Our ILP model is slightly different from the one in [15], since it reflects the objective of certain hospital managers [1]. In [15] the objective function considers the costs of performing the surgery on a certain day and the costs of not performing it at all. Whereas, we assign a *score* to each surgical case in the waiting list, which accounts for two issues, namely the time spent since the decision date and the specific relevance of the surgery as in [1]. The objective is to determine MSS and SCA in order to maximize the score of the cases selected for the next time horizon. While finding the optimal solution to this integrated MSS+SCA problem may be time-consuming and may require significant computational resources, we will show that an efficient decomposition approach produces very good solutions in a fraction of computational time. This is especially true when considering medium-to-large operating theaters, since computational times of integrated models may grow very fast.

The plan of the paper is as follows. In Section 2 the considered model is described in detail. In Section 3 the mathematical formulations and the algorithms are formally introduced. Computational experiments are illustrated and discussed in Section 4. Finally, in Section 5 some conclusions are drawn.

2 Problem description

This study focuses on the problem of allocating elective surgeries to operating rooms over a given time horizon (e.g., one week) covering the problems denoted as MSSP and SCAP.

All elective surgeries are grouped into *surgical disciplines* (e.g., orthopedics, day surgery). The main input to the overall planning problem is the *waiting list* of each discipline, containing all the case surgeries that currently need to be performed. For each case surgery, the following information is specified in the waiting list:

- Surgery code – identifies the specific type of surgery.
- Processing time – expected duration of the surgery (including setup times due to cleaning and OR preparation for the next surgery). We assume all these times to be deterministic; i.e., they are not affected by uncertainty.
- Decision time – date when the surgery entered the waiting list (i.e., the general practitioner recommended the surgical treatment).
- Waiting time – days currently elapsed since the decision time.
- Priority class – surgeries are classified in three priority classes A, B or C (A being the most urgent), according to the regulatory essential assistance levels.
- Due date – nominal time within which the surgery should be performed. It is set on the basis of the entrance time and the priority class.

Cases are carried out in full-day *OR sessions* each assigned to a single surgical discipline and the total processing time of the surgeries allocated to each session must not exceed the session duration minus a given *slack*. The slack is used to absorb possible delays and to reduce the chances of surgeons overtime.

In general, a MSS may be subject to various types of restrictions, which must be accounted for when planning:

- *discipline-to-OR restrictions.* Certain disciplines can only be performed in a restricted set of ORs, due to size and/or equipment constraints.
- *Limits on discipline parallelism.* Typically, no more than k OR sessions of a certain discipline can take place at the same time, e.g., because only k equipes for that discipline are available.
- *OR sessions-per-discipline restrictions.* Lower and upper limits to the number of OR sessions assigned to each discipline throughout one week can be specified; e.g., because of workload balancing or other management rules.
- *OR reservation.* The hospital management may reserve one or more OR sessions to certain disciplines every day.

The objectives of the overall problem are twofold:

- Resource perspective – OR sessions capacity and actual demand should be matched as much as possible, since both under- and over-utilization of an operating room are wasteful;
- Patient perspective – In organizational terms, the quality of service is expressed by the due date performance of the service, which in turn is related to having cases done within the respective due dates as much as possible.

In general, these two objectives may be partially conflicting. Since tightly filling OR sessions may ignore surgeries' due dates, and scheduling surgeries based on their due date only may lead to inefficient utilization of ORs. Hence, we define an objective function that allows to account for both these aspects. Namely, we define the *score* of each surgery as the product of the surgery processing time and a coefficient which depends on how close is the surgery to its due date. The problem is to decide the MSS and the SCA so that the total score of selected cases is maximized.

3 Formulations and decomposition approach

In what follows, we first present notation and a mathematical programming formulation (Section 3.1), and thereafter we present the decomposition approaches proposed to solve MSSP+SCAP (Section 3.2).

3.1 Notation and formulation

In what follows, we denote by S the set of surgical disciplines (indexed by s), and by I_s the set of surgeries (indexed by i) in the waiting list of the surgical discipline s. The expected duration of the i-th surgery of discipline s is P_{is}. Let N_{OT} be the total number of sessions available for planning in one week in the operating theater and J the set of operating rooms, indexed by r. All sessions have length O^{max} (time). Weekdays are indexed by $w \in \{1, \ldots, 5\}$, from Monday ($w = 1$) to Friday ($w = 5$) and we denote by S_s^{min} and S_s^{max} the minimum and, respectively, maximum number of OR sessions to be allocated to the surgical discipline s in one week. The maximum number of parallel sessions that can be assigned to discipline s is denoted by PS_s, while NA_s is the set of operating rooms not available for discipline s.

For the i-th surgery of discipline s, we define a *score* as $K_{is} = P_{is}(W - R_{is})$, where W is the maximum allowed waiting time for low-priority surgeries, and R_{is} are the days to due date (this number is negative for late surgeries).

The overall objective is to select a set of surgeries to be performed that maximizes the overall score. Notice that using the maximum allowed waiting time for low-priority surgeries W guarantees that urgent surgeries have greater impact on the objective function and thus the model has more incentives to select them.

We next introduce a mathematical programming model of the above problem. In the model, $x_{isrw} = 1$ if the i-th surgery of surgical discipline s is assigned to OR r for the day w, and $y_{srw} = 1$ if the surgical discipline s is assigned to OR r the day w.

$$\max \quad \sum_s \sum_i \sum_r \sum_w K_{is} \cdot x_{isrw} \tag{1}$$

$$\sum_r \sum_w x_{isrw} \leq 1 \quad \forall i, s \tag{2}$$

$$\sum_i P_{is} \cdot x_{isrw} \leq O^{max} \cdot y_{srw} \quad \forall s, r, w \tag{3}$$

$$\sum_w \sum_s y_{srw} \leq 1 \quad \forall r \tag{4}$$

$$\sum_w \sum_r y_{srw} \geq S_s^{min} \quad \forall s \tag{5}$$

$$\sum_w \sum_r y_{srw} \leq S_s^{max} \quad \forall s \tag{6}$$

$$\sum_r y_{srw} \leq PS_s \quad \forall w, s \tag{7}$$

$$\sum_w \sum_{r \in NA_s} y_{srw} = 0 \quad \forall s \tag{8}$$

$$x_{isrw} \in \{0, 1\} \quad \forall i, s, r, w \tag{9}$$

$$y_{srw} \in \{0, 1\} \quad \forall s, r, w \tag{10}$$

Constraint (2) states that each surgery can be performed at most once. Constraint (3) establishes an upper limit to the duration of the surgical cases assigned to the same session. Constraints (4) guarantee that there are no two surgical disciplines assigned to the same OR at the same time. Constraint (5–6) bound the number of weekly OR sessions assigned to each discipline. Constraints (7) limit the number of parallel sessions assigned to the same surgical discipline. Discipline-to-OR restrictions are taken into account by constraint (8).

3.2 Decomposition approach

The mathematical models introduced in the previous sections may in general require large computation times, even to find suboptimal solutions. Therefore, in order to quickly reach good solutions for MSSP+SCAP, we propose an efficient decomposition approach. The idea is to address MSSP and SCAP sequentially by first producing an MSS and next, given the MSS as input, determining the SCA.

3.2.1 Solving the MSSP

The algorithm that produces the MSS works as follows:

(i) given the waiting list of each surgical discipline as input, for each discipline a number of *candidate sets* is quickly generated. Each candidate set consists of a set of surgeries such that the sum of their processing times does not exceed O^{max}.

(ii) a complete surgical case assignment is temporarily produced by filling available sessions during the week with candidate sets.

(iii) surgical cases are discarded, and only the MSS is retained.

In step (i), in order to generate the candidate sets, we consider the problem of filling a number of bins (corresponding to candidate sets) of given capacity (the OR session length) with items (the surgical cases), each having a given size (the expected processing time of the surgical case) and a given value (the score associated to the surgery). We consider S_s^{max} bins of capacity O^{max}. Next, for each surgical case we compute the ratio between score K_{is} and processing time P_{is}, and we order all the surgical cases of discipline s by non-increasing values of such ratios. Hence, we sequentially fill the bins according to the first-fit rule, i.e., assigning the current item to the first bin that fits. In this way, we generate S_s^{max} candidate sets. For the f-th candidate set of discipline s, call it c_{sf}, we compute the *values* v_{sf} as the sum of the scores K_{is} of the surgeries in c_{sf}.

In step (ii), we select a subset of the candidate sets generated, to produce a feasible plan. This problem can be formulated as a min-cost flow problem (Figure 1) on a suitable network $\mathcal{N}(V, A)$, in which the flow in the network represents the assignment of candidate sets to OR sessions. We next describe the structure of the network in detail.

The network $\mathcal{N}(V, A)$ has one source node Q and one sink node R that respectively generate and absorb N_{OT} units of flow. The remaining nodes do not produce or absorb flow, and are divided into six different layers, namely: *discipline* nodes s ($s = 1, \ldots, |S|$), *candidate* nodes c_{sf} ($s = 1, \ldots, |S|$, $f = 1, \ldots, S_s^{max}$), *in-day* nodes (s, w) ($s = 1, \ldots, |S|$, $w = 1, \ldots, 5$), *out-day* nodes (s', w') ($s' = 1, \ldots, |S|$, $w' = 1, \ldots, 5$), *room-day* nodes (r, w) ($r = 1, \ldots, |J|$, $w = 1, \ldots, 5$) and *room* nodes r ($r = 1, \ldots, |J|$). An arc from node i to node j will be denoted by $[i, j]$.

From the source node Q, there are $|S|$ outgoing arcs $[Q, s]$, one for each discipline node. These arcs have a lower bound on the flow equal to S_s^{min}. There is no upper capacity nor cost for these arcs. The lower bound on the flow of these arcs guarantees that at least S_s^{min} sessions are assigned to discipline s.

From each discipline node s, there are S_s^{max} outgoing arcs to candidate nodes c_{sf}, each corresponding to a candidate set generated for discipline s. All these arcs have no cost and they all have capacity 1, which guarantees that each candidate set is assigned at most once.

From each candidate node c_{sf} there are 5 outgoing arcs (one for each weekday), connecting c_{sf} to all *in-day nodes* (s, w), $w = 1, \ldots, 5$. These arcs have no upper capacity, however the flow on arcs $[c_{sf}, (s, w)]$ cannot be greater than 1, since at most one unit of flow enters each candidate node c_{sf}. On each of these 5 arcs, the cost associated to the flow is $-v_{sf}$, meaning that v_{sf} is actually gained if candidate session c_{sf} is selected.

Next, there is an arc from each in-day node (s, w) to the respective out-day node (s', w') with $s' = s$ and $w' = w$. The upper capacity of these arcs guarantees that no more than PS_s parallel sessions are assigned to discipline s in one day.

Each out-day node (s', w') has one outgoing arc to each corresponding room-day node (r, w) (i.e., such that $w = w'$ and discipline s can be performed in room r). No costs or capacities are associated to these arcs.

From each room-day node (r, w) there is an arc $[(r, w), r]$ to a room node r. Hence, each room node r has 5 ingoing arcs. These arcs have no cost and capacity 1, thus enforcing that at most one session is assigned to each room on each day.

Finally, there is one outgoing arc from each room node r to the sink node R with no associated cost or capacity.

The objective function of the min-cost flow model corresponds to maximizing the sum of the values v_{sf} of selected candidate sets, i.e., assigning to the MSS the most profitable candidate sets.

We next show that the solution to the min-cost flow problem generates a surgical case assignment that satisfies the constraints of model (1–10). Compliance with constraints (2) and (3) is ensured by the generation procedure of step (i), which assigns each surgery at most once and produces candidate sets of total duration not exceeding O^{max}. Constraints (4) are enforced by the capacities of arcs $[r, R]$ from room nodes to the sink. The capacities of arcs $[Q, s]$ from the source to session nodes take care of the constraint (5) on the minimum number of sessions. The constraint (6) on the maximum number of sessions is enforced by the fact that we only generate S_s^{max} candidate sets for each discipline s in the procedure of step (i). The constraint (7) on the maximum number of parallel sessions for a discipline s is enforced by the capacity of arcs $[(s,w),(s',w')]$ from in-day to out-day nodes. Finally, discipline-to-OR constraints (8) are taken into account by not generating forbidden arcs between out-day nodes and room-day nodes.

In Figure 1 an example of an instance with two disciplines and two ORs is shown. For the sake of clarity, arc capacities are not shown when they are not binding. The only nonzero cost arcs are those connecting candidate nodes to in-day nodes. Note that all arcs outgoing a node c_{sf} have the same negative cost $-v_{sf}$. Observe that in this example, discipline 2 cannot be performed in OR 1, hence the arcs from out-day nodes (s',w') to room-day nodes (r,w) are missing for $s'=2$, $r=1$ and $w=w'$.

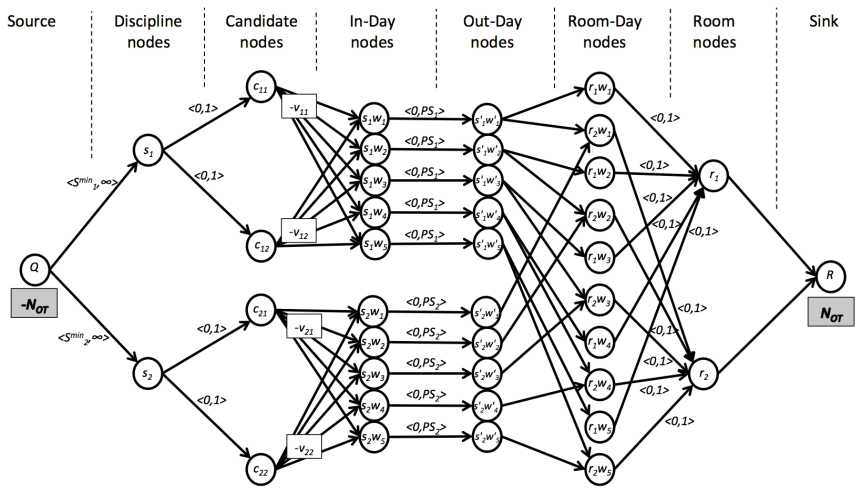

Fig. 1 Example of the min-cost flow formulation

Finally, in step (iii), once the min-cost flow problem is solved, the MSS structure is defined by the nonzero flows in the arcs connecting out-day and room-day nodes, i.e., arcs $[(s',w'),(r,w)]$ with $w=w'$.

3.2.2 Solving the SCAP

It can be observed that solving the SCAP corresponds to solving s independent multiple-knapsack problems [13], one for each surgical discipline s, where surgeries correspond to items and sessions to knapsacks. Each multiple-knapsack problem corresponds to the following model for a given discipline s:

$$\max \quad \sum_h \sum_i K_{is} \cdot x_{ish} \tag{11}$$

$$\sum_h x_{ish} \leq 1 \quad \forall i \tag{12}$$

$$\sum_i P_{is} \cdot x_{ish} \leq O^{max} \quad \forall h \tag{13}$$

$$x_{ish} \in \{0, 1\} \quad \forall i, h \tag{14}$$

The decision variables are $x_{ish} = 1$ if the i-th surgery of discipline s is assigned to the h-th OR session of discipline s; otherwise $x_{ish} = 0$.

4 Case study and computational results

In this section we discuss the computational experiments set up for evaluating the proposed decomposition approach against an integrated mathematical model. More specifically, in Section 4.1 we introduce the benchmark instances and in Section 4.2 we illustrate the numerical results obtained.

4.1 Experimental design and setting

In our computational experiments, we generated 6 different sets of benchmark instances. This is done by considering 3 different OT sizes (5, 10 and 15 ORs) and, for each OT size, 2 different waiting lists by considering 200 or 300 surgical cases for each OR. Each set is composed by 10 instances. More in detail, we refer to these sets as ($|J|$,β), where $|J|$ is the OT size and β the multiplier used to obtain the waiting lists (i.e., $(5, 200)$ refers to instances with 5 ORs and having $1000 = 5 \cdot 200$ surgical cases in the waiting lists). More precisely, we were provided by the San Giuseppe hospital of Empoli with the current waiting lists I_s for the six surgical disciplines in S. The waiting lists of all the specialties in our benchmark instances are obtained through nonparametric bootstrapping [11], which consists in sampling — with replacement — $\sum n_s$ surgeries from the set $\bigcup I_s$. Therefore, the size of the waiting list of each discipline is variable even though the total number of surgical cases is $|J| \cdot \beta$. The maximum allowed waiting time for less urgent surgeries is $W = 90$ days.

The operating rooms are all identically equipped, even though two of them (say, $r = 4, 5$) are bigger than the others. Benchmark sets with 10 and 15 operating rooms are obtained by replicating each OR two or three times respectively. Focusing on elective surgeries only, the weekly surgery plan spans five days, from Monday to Friday. Each session lasts 11.5 hours. However, in order to account for possible delays and/or uncertainties affecting surgery duration, we introduce a planned slack time of 60 minutes. The duration of the session is divided into time units of 15 minutes each, thus resulting in $O^{max} = 42$.

Six different disciplines have been considered: gynecology ($s =$ GYN), general surgery ($s =$ GS), otolaryngology ($s =$ ENT), urology ($s =$ URO), day surgery ($s =$ DS) and orthopedic surgery ($s =$ ORTH).

Table 1 Minimum and maximum number of OR sessions in one week.

	$\lvert J\rvert = 5$		$\lvert J\rvert = 10$		$\lvert J\rvert = 15$	
	S_s^{min}	S_s^{max}	S_s^{min}	S_s^{max}	S_s^{min}	S_s^{max}
GYN	2	6	4	12	6	18
GS	6	10	12	20	18	30
ENT	1	5	2	10	3	15
URO	1	5	2	10	3	15
DS	3	7	6	14	9	21
ORTH	6	10	12	20	18	30

Some disciplines have a set of non-available ORs. That is, gynecology surgeries should always be performed in OR $r = 1$ ($NA_{\text{GYN}} = \{2, 3, 4, 5\}$), and orthopedic surgeries have to be performed either in room $r = 4$ or $r = 5$. Thus, $NA_{\text{ORTH}} = \{1, 2, 3\}$. Nevertheless, these ORs are not exclusively assigned to these disciplines. With $|J| = 10$ ($|J| = 15$) the number of unavailable rooms is duplicated (triplicated). General surgery and orthopedic surgery allow 2 parallel OR sessions ($PS_{\text{GS}} = 2$, $PS_{\text{ORTH}} = 2$), whereas all the other surgical disciplines do not admit parallel sessions. When considering larger instances with 10 rooms, the limit is set to 4 for general surgery and orthopedic and to 2 for all other specialities. Analogously, when $|J| = 15$ the limit is set to 6 and 3, respectively. The values of S_s^{min} and S_s^{max} are given in Table 1.

Since the model (1–10) presents symmetries due to the presence of identical rooms, we add a set of symmetry-breaking constraints (15) having the only purpose of speeding up computations. Given an arbitrary order among the disciplines s and rooms r, these constraints guarantee that rooms are assigned to disciplines in this order. For instance, if discipline s has been assigned to the operating room r, then none of the rooms $1, 2, \ldots, r-1$ can be assigned to discipline l in the same day, if $l > s$.

$$y_{srw} + \sum_{k=1}^{r-1} y_{lkw} \leq 1 \qquad \forall s, l \in \{s+1, \ldots, |S|\}, r, w \tag{15}$$

Notice that in our case study, symmetry-breaking constraints are compatible with constraints (8) on discipline-to-OR restrictions. In general, this may not be true.

Tests have been performed on a 3.2 GHz Intel Core i3 processor with 4 GB of RAM, using OPL Studio 6.1 and the CPLEX 11.2 MILP solver for the mathematical programming models. The maximum computation time has been set to 60 minutes for each optimization run of the integrated model, whereas the decomposition approaches are allowed to run for at most 60 seconds to solve MSSP+SCAP.

4.2 Numerical results

We now discuss the performance of the proposed decomposition approach on benchmark instances. The main results are presented in Table 2. The first column in the table shows the name of the benchmark set ($|J|$,β). The next 5 columns report the performance indices of the integrated mathematical model, while in the last 5 columns we report the same performance indices for the decomposition approach. More specifically, Columns 2 and 7 show the values of the objective function. Columns 3 and 8 show the optimality gap of the two approaches, computed with respect to the best upper bound found by the integrated model. Columns 4 and 9 report the number of empty time

Table 2 Comparison between integrated and decomposition approaches

	Integrated Model					Decomposition approach				
($\|J\|$,β)	objective function	gap	empty t.u.	scheduled surgeries	CPU time	objective function	gap	empty t.u.	scheduled surgeries	CPU time
(5,200)	205610.9	0.36%	1.7	176.7	3600.0	205325.2	0.50%	1.8	177.8	14.01
(10,200)	411257.3	0.34%	2.5	364.4	3600.0	411013.8	0.40%	4.7	364.6	19.29
(15,200)	621655.4	0.38%	11.7	535.5	3600.0	621803.8	0.35%	15.5	534.8	29.33
(5,300)	218394.3	0.41%	2.2	179.7	3600.0	218282.9	0.46%	1.9	180.2	19.26
(10,300)	437185.2	0.33%	1.3	354.6	3600.0	436610.3	0.46%	4.4	355.9	25.28
(15,300)	651406.3	0.45%	28.7	545.6	3600.0	650779.8	0.54%	8.6	541.3	30.94

units across the whole week. An empty time unit is a 15-minute not assigned to any surgery. Note that the total number of available time units is 1050, 2100 and 3150 for instances having 5, 10 and 15 operating rooms, respectively. Columns 5 and 10 indicate the total number of surgeries scheduled in the week. Finally, Columns 6 and 11 report the CPU time needed by the two approaches. Each row of the table reports the average values over ten instances belonging to the same benchmark set. A few comments are in order.

- No overall big differences emerge between the quality of the solutions provided by the integrated and the proposed decomposition approaches. More specifically, in terms of objective function the two algorithms provide comparable solutions, with a slightly better performance of the integrated approach. The achieved value of the objective function displays a linear increase as the number of available ORs increases.
- In both approaches, the optimality gap is very small (less than 0.5% on average) and almost independent of instance size.
- As for the percentage of empty time units, both the approaches provide an efficient use of the OT. In fact, the empty time units never exceed 1% of total available time units, even in largest instances. This shows how both the integrated and the decomposition approaches are able to effectively exploit the available OT capacity. Also the total number of surgeries is comparable between the two approaches, showing a linear increase in $|J|$.
- Although the quality of the solutions provided by the two approaches is similar, relevant differences arise when considering CPU times. In fact, the integrated approach was always truncated after one hour of computation time, whereas the proposed decomposition approach only required few seconds to obtain solutions of comparable quality.

5 Conclusions and future research

In this paper we proposed a heuristic decomposition approach to determine the master surgical schedule and a surgical case assignment (MSS+SCA).

Mathematical programming models for tackling MSSP+SCAP are able to provide optimal or near-optimal solutions, but their computation times are high. For this reason, we propose a heuristic decomposition approach able to provide near-optimal solutions in very small computation time. The approach is based on a two-phase decomposition in which first MSSP is solved as a minimum cost flow problem, and then SCAP is solved as a multiple-knapsack problem. We tested the decomposition approach on several realistic instances of various OT and waiting list sizes. The results show that the decomposition approach is very effective in practice, reducing the computation time by two orders of magnitude while maintaining solutions close to optimality. Hence, the proposed decomposition scheme can also be embedded in a decision support system, as a tool to perform what-if analysis, or to recompute feasible plans in the face of unpredicted events.

Future research may address possible refinements and improvements of the models and algorithms presented, such as:

- extending the model to consider detailed surgeons' timetables and bed occupancy,
- including uncertainties (e.g., in surgical case durations) in the model,
- evaluate even faster approaches in which SCAP is solved by means of a fast heuristic.

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